Light Quark Masses and the CP violation parameter ϵ'/ϵ

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We present estimates of light quarks masses using lattice data. Our main results are based on a global analysis of all the published data for Wilson and Staggered fermions, both in the quenched approximation and with $n_f = 2$ dynamical flavors. The Wilson and Staggered results agree after extrapolation to the continuum limit for both the $n_f = 0, 2$ theories. Our best estimates, in the $\overline{MS}$ scheme at scale 2 GeV, are $\overline{m} = 3.2(4)$ MeV and $m_s = 90(20)$ MeV in the quenched approximation, and $\overline{m} \sim 2.7$ MeV and $m_s \sim 70$ MeV for the $n_f = 2$ theory. These estimates are significantly smaller than phenomenological estimates based on sum rules, but maintain the ratios predicted by chiral perturbation theory (χPT). Along with the new estimates of 4-fermion operators, lower quark masses have a significant impact on the extraction of ϵ'/ϵ from the Standard Model.

1. LIGHT QUARK MASSES

The masses of light quarks m_u, m_d, and m_s are three of the least well known parameters of the Standard Model. These quark masses have to be inferred from the masses of low lying hadrons. χPT relates the masses of pseudoscalar mesons to m_u, m_d, and m_s, however, the presence of the unknown scale μ in $\mathcal{L}_{\chi\mathrm{PT}}$ implies that only ratios of quark masses can be determined. For example $2m_s/(m_u + m_d) \equiv m_s/\overline{m} = 25$ at lowest order, and 31 at next order [1,2]. Latest estimates using QCD sum rules give $m_u + m_d = 12(1)$ MeV [3]. However, as discussed in [4], a reanalysis of sum rules shows that far more experimental information on the hadronic spectral function is needed before sum rules can give reliable estimates. Thus, lattice QCD is currently the most promising approach.

To extract a, $\overline{m}$, m_s we fit the global data as

$$
\begin{aligned}
M_{PS} &= B_{PS}(m_1 + m_2)/2 \\
M_V &= A_V + B_V(m_1 + m_2)/2 \ , \qquad (1)
\end{aligned}
$$

for each value of the lattice parameters, β, n_f, fermion action. From B_{PS}, A_V, B_V we determine the three desired quantities; the lattice scale a using M_ρ, $\overline{m}$ using M_π^2/M_ρ^2, and m_s in three different ways using M_K, M_{K^*}, M_ϕ. Throughout the analysis we assume that ϕ is a pure $s\overline{s}$ state.

*We acknowledge the support of the ACL at Los Alamos and NCSA at Urbana for computational support.

Note that using Eq. 1 means that we can predict only one independent quark mass from the pseudoscalar data, which we choose to be $\overline{m}$. The reason for this truncation is that in most cases the data for M_π and M_ρ exist at only $2-4$ values of "light" quark masses in the range $0.3m_s - 2m_s$. In this restricted range of quark masses the existing data do not show any significant deviation from linearity. One thus has to use M_V in order to extract m_s. Details of our analysis and of the global data used are given in [5].

For Wilson fermions the lattice quark mass, defined at scale q^*, is taken to be $m_L(q^*)a = (1/2\kappa - 1/2\kappa_c)$. For staggered fermions $m_L(q^*) = m_0$, the input mass. The $\overline{MS}$ mass at scale μ is $m_{\overline{MS}}(\mu) = Z_m(\mu a) m_L(a)$, where Z_m is the mass renormalization constant relating the lattice and the continuum regularization schemes at scale μ, and $\lambda = g^2/16\pi^2$. In calculating Z_m, *a la* Lepage-Mackenzie, we use $\alpha_{\overline{MS}}$ for the lattice coupling, use "horizontal" matching, *i.e.* $\mu = q^* = 1/a$, and do tadpole subtraction. We find that the results are insensitive to the choice of q^* in the range $0.86/a - \pi/a$ and to whether or not tadpole subtraction is done. Once $m_{\overline{MS}}(\mu)$ has been calculated, its value at any other scale Q is given by the two loop running. We quote all results at $Q = 2$ GeV.

We extrapolate the lattice masses to $a = 0$ using the lowest order corrections (Wilson are $O(a)$ and Staggered are $O(a^2)$). In the quenched fits

Table 1
Summary of results in MeV in $\overline{MS}$ scheme at $\mu = 2$ GeV. The label $W(0)$ stands for Wilson with $n_f = 0$. An additional uncertainty of $\sim 10\%$ due to the uncertainty in the lattice scale a is suppressed.

	$\overline{m}$	$m_s(M_K)$	$m_s(M_\phi)$	$m_s(M_K^*)$
$W(0)$	3.3(4)	83(10)	96(10)	76(20)
$S(0)$	3.1(1)	78(3)	96(2)	87(2)
$W(2)$	2.5(3)	63(8)	77(10)	78(22)
$S(2)$	2.9(3)	73(8)	66(6)	59(6)

we omit points at the stronger couplings ($a > 0.5$ GeV^{-1}) because we use only the leading correction in the extrapolation to $a = 0$, and because the perturbative matching becomes less reliable as β is decreased. The bottom line is that we find that the leading corrections give a good fit to the data, and in the $a = 0$ limit the two different fermion formulations give consistent results. Our final results are summarized in Table 1.

The global data for $\overline{m}$ and the extrapolations to $a = 0$ for Wilson are shown in Fig. 1. Using the average of quenched estimates given in Table 1 we get

$$\overline{m}(\overline{MS}, 2\ \text{GeV}) = 3.2(4)(3)\ \text{MeV} \quad \text{(quenched)}. (2)$$

where the first error estimate is the larger of the two extrapolation errors, and the second is that due to the uncertainty in the scale a.

The pattern of $O(a)$ corrections in the unquenched data ($n_f = 2$) is not clear and we only consider data for $\beta \geq 5.4$. The strongest statement we can make is qualitative; at any given value of the lattice spacing, the $n_f = 2$ data lies below the quenched result. Taking the existing data at face value, we find that the average of the Wilson and staggered values are the same for the choices $\beta \geq 5.4$, $\beta \geq 5.5$, or $\beta \geq 5.6$. We therefore take this average

$$\overline{m}(2\ \text{GeV}) \approx 2.7\ \text{MeV} \qquad (n_f = 2\ \text{flavors})\ , \quad (3)$$

as the current estimate. To obtain a value in the physical case of $n_f = 3$, the best we can do is

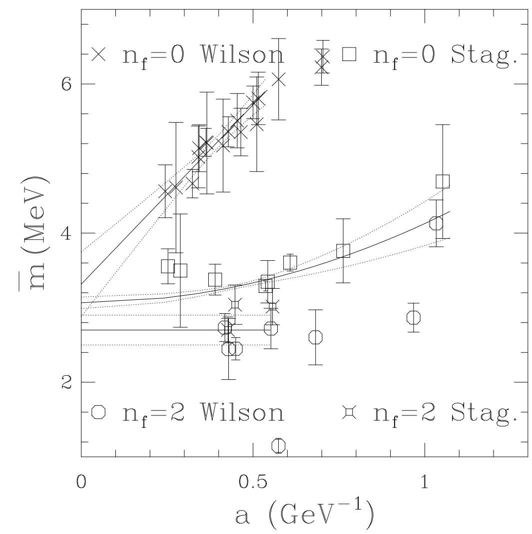

Figure 1. $\overline{m}(\overline{MS}, 2\ \text{GeV})$ extracted using M_π data with the scale set by M_ρ.

to assume a behavior linear in n_f. In which case extrapolating the $n_f = 0$ and 2 data gives

$$\overline{m}(2\ \text{GeV}) \approx 2.5\ \text{MeV} \qquad (n_f = 3\ \text{flavors}). \quad (4)$$

We stress that this extrapolation in n_f is extremely preliminary.

We determine m_s using the three different mass-ratios, M_K^2/M_π^2, M_{K^*}/M_ρ, and M_ϕ/M_ρ. Using a linear fit to the pseudo-scalar data constrains $m_s(M_K) = 25\overline{m}$. Using the vector mesons M_K^* and M_ϕ gives independent estimates. The quenched data and extrapolation to $a = 0$ of $m_s(M_\phi)$ are shown in Fig. 2. The average of Wilson and staggered values are $m_s(M_\phi) = 96(10)$ MeV and $m_s(M_{K^*}) = 82(20)$ MeV where the errors are taken to be the larger of Wilson/staggered data. From these we get our final estimate

$$m_s = 90(15)\ \text{MeV} \qquad \text{(quenched)}\ . \quad (5)$$

The $n_f = 2$ data shows a pattern similar to that for $\overline{m}$. Therefore, we again take the average of values quoted in Table 1 to get

$$m_s = 70(11)\ \text{MeV} \qquad (n_f = 2)\ . \quad (6)$$

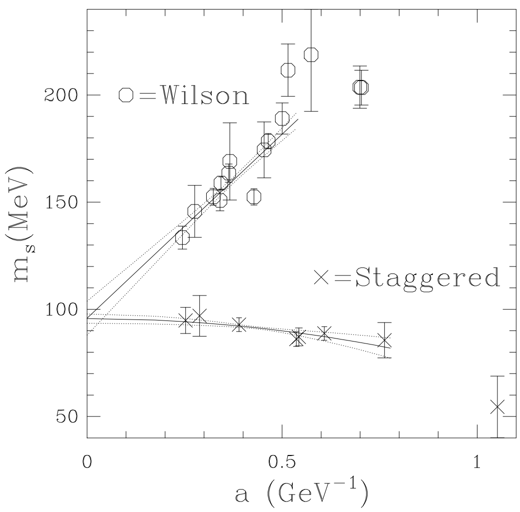

Figure 2. Comparison of $m_s(\overline{MS}, 2\,\mathrm{GeV})$ extracted using M_ϕ for the quenched Wilson and staggered theories.

The error estimate reflects the spread in the data.

Qualitatively, the data show three consistent patterns. First, agreement between Wilson and Staggered values. Second, for a given value of a the $n_f = 2$ results are smaller than those in the quenched approximation. Lastly, the ratio $\overline{m}/m_s(M_\phi)$ is in good agreement with the next-to-leading-order predictions of chiral perturbation theory for both the $n_f = 0$ and 2 estimates. It is obvious that more lattice data are needed to resolve the behavior of the unquenched results. However, the surprise of this analysis is that both the quenched and $n_f = 2$ values are small and lie at the very bottom of the range predicted by phenomenological analyses [1].

2. CP VIOLATION and ϵ'/ϵ

A detailed analysis of 4-fermion matrix elements with quenched Wilson fermions at $\beta = 6.0$ is presented in [6]. The methodology, based on the expansion of the matrix elements in powers of the quark mass and momentum, is discussed in [7]. Our estimates in the NDR scheme at $\mu = 2$ GeV are

$$\begin{aligned} B_K &= 0.68(4)\ , \\ B_D &= 0.78(1)\ , \\ B_7^{3/2} &= 0.58(2)\ , \\ B_8^{3/2} &= 0.81(3)\ . \end{aligned} \tag{7}$$

The errors quoted are statistical. The major remaining sources of errors in these estimates are lattice discretization and quenching.

To exhibit the dependence of the Standard Model (SM) prediction of ϵ'/ϵ on the light quark masses and the B parameters we write

$$\epsilon'/\epsilon = A\left(c_0 + c_6 B_6^{1/2} M_r + c_8 B_8^{3/2} M_r\right)\ , \tag{8}$$

where $M_r = (158\,\mathrm{MeV}/(m_s + m_d))^2$. For reasonable choices of SM parameters Buras *et al.* estimate $A = 1.3 \times 10^{-4}$, $c_0 = -1.3$, $c_6 = 7.9$, $c_8 = -4.3$ [8]. Thus, to a good approximation $\epsilon'/\epsilon \propto M_r$; and increases as $B_8^{3/2}$ decreases. As a result, our estimates of $\overline{m}, m_s, B_8^{3/2}$ increase ϵ'/ϵ by roughly a factor of three compared to previous analysis, *i.e.* from 3.6×10^{-4} to $\sim 10.4 \times 10^{-4}$. This revised estimate lies in between the Fermilab E731 ($7.4(5.9) \times 10^{-4}$) and CERN NA31 ($23(7) \times 10^{-4}$) measurements and provides a scenario in which direct CP violation can be explained within the Standard Model.

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